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Nonlinear magneto-acoustics of tetragonal antiferromagnets

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Abstract. The results of theoretical study of the influence of mechanical stress, electric and magnetic fields on the conditions for magneto-elastic solitons existence and their dynamic properties in the symmetric phase of a tetragonal antiferromagnet are presented. The conditions for the occurrence of nonlinear magneto-elastic resonance of long and short waves (which is analogous to the Zakharov–Benney triad resonance) in this system are derived. The possibility of experimental detection of this new phenomenon is discussed.

1. Introduction

Recent years have seen an intensive study of various properties of magnets with a magneto-electric interaction. Electric and magnetic properties in these magnets are interconnected: using an electric field one can control magnetic properties, and *vice versa*, a magnetic field can be used to control electric properties [1]. A magneto-electric interaction gives rise to the appearance of the magnetization $\boldsymbol{m}$, induced by the electric field $\boldsymbol{E}$, or the electric polarization $\boldsymbol{P}$, induced by the magnetic field $\boldsymbol{H}$. The most fully studied is the linear magneto-electric effect in antiferromagnets (AFMs), this effect being explained (if considered phenomenologically) by the terms of the lmP type in the expression for the free energy density ($\boldsymbol{l}$ is the antiferromagnetism vector). In this case $P \sim H$, $m \sim E$. The proportionality factor may reach 10^{-2} and more.

The peculiarities of acoustic properties and antiferromagnetic resonance in tetragonal magneto-electric AFM was investigated by Turov *et al* [2]. It has been shown that the values of magneto-electric largely depend on the mutual orientation of the electric and the magnetic fields, on the magnetic anisotropy type and on the value and the sign of the magnetic anisotropy constant in the basal plane. For example, the effect of the acoustic double refraction (lifting the degeneracy of transverse waves) due to the magneto-electric effect may reach several per cent. The conditions at which new effective nonlinear moduli of elasticity appear, which are non-zero only in the case when the sample is simultaneously affected by both magnetic and electric fields, was identified by Menshenin and Turov [3]. It is pointed out in [2, 3] that magneto-electric effects are enhanced in the vicinity of magnetic orientational phase transitions. However, for the latter to be achieved, sufficiently strong electric and magnetic fields need to be applied. It has also been pointed out that a possibility of a wide variety of nonlinear effects in centre-antisymmetric AFMs.

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Nonlinear magneto-elastic waves in magnets have been discussed without considering magneto-electric interaction, for example, in [4–7]. It is well known that with the help of directed pressure the point of the orientational phase transition can be approached, which may amplify the manifestation of the magneto-electric effect. This paper reports on research of the effects of mechanical stress, the electric and magnetic fields, on the stability of magneto-elastic solitons and on characteristics on nonlinear magneto-elastic resonance of long and short waves (which is analogous to the Zakharov–Benney triad resonance) in tetragonal AFMs with an 'easy-plane' anisotropy. The conditions of the Zakharov–Benney resonance of long and short waves (long–short-wavelength resonance) are satisfied if the group velocity of a short (spin) wave is equal to the phase velocity of the long (elastic) wave. The long and short waves become coupled, and as a result of this slow oscillations, arising from the beating in a spin wave, lead to the excitation of an elastic wave. An excitation of lower frequencies by higher ones takes place, and *vice versa*. The solitons arising in this system originate from the energy exchange between spin and elastic waves. It should be pointed out that the linear magneto-elastic resonance (the intersection of the dispersion curves for spin and elastic waves) is pronounced most clearly in the frequency range, where the elastic wave length is close to that of the spin wave. In the vicinity of the intersection point the combined magneto-elastic waves appear, which lead to the intensive dissipation and anomalous dispersion of the sound wave.

The present paper consists of five sections. In section 2 the ground state is examined and fundamental equations of weakly nonlinear dynamics are obtained. Section 3 looks into peculiarities of nonlinear dynamics brought about by the interaction of the Goldstone modes, whose evolution equation is the modified Kortweg–de Vries equation (MKdV). Section 4 focuses upon the interaction of the activation waves, whose evolution is described by the nonlinear Schrödinger equation (NSE), as well as the Zakharov–Benney resonance.

2. The ground state and the equations of weakly nonlinear dynamics

Let us consider an unbounded tetragonal double-sublattice AFM with an 'easy-plane'-type anisotropy ($K > 0$). When deriving fundamental equations we shall proceed from the free energy density F, which shall contain a magnetic, electropolarization, elastic, magneto-elastic and magneto-electric contributions [3]:

$$
\begin{aligned}
F_m &= 2M_0H_E\boldsymbol{m}^2 - 2M_0\boldsymbol{m}\boldsymbol{H} + \frac{1}{2}Kl_z^2 + \frac{1}{2}K_2l_x^2l_y^2 + \frac{1}{2}E_l'a_0^2\left(\frac{\partial \boldsymbol{l}}{\partial x_k}\right)^2 + \frac{1}{2}E_m'a_0^2\left(\frac{\partial \boldsymbol{m}}{\partial x_k}\right)^2 \\
&\qquad - M_0\boldsymbol{m}\boldsymbol{H}_m \\
F_p &= \frac{1}{2\kappa_\perp}(P_x^2 + P_y^2) + \frac{1}{2\kappa_\parallel}P_z^2 - \boldsymbol{P}\boldsymbol{E} \\
F_e &= \tfrac{1}{2}C_{11}(e_{xx}^2 + e_{yy}^2) + C_{12}e_{xx}e_{yy} + C_{13}(e_{xx} + e_{yy})e_{zz} + \tfrac{1}{2}C_{33}e_{zz}^2 + 2C_{44}(e_{xz}^2 + e_{yz}^2) \\
&\qquad + 2C_{66}e_{xy}^2 - \sigma_{ik}e_{ik} \\
F_{le} &= B_{11}(l_x^2e_{xx} + l_y^2e_{yy}) + B_{12}(l_x^2e_{yy} + l_y^2e_{xx}) + B_{13}(e_{xx} + e_{yy})l_z^2 + B_{31}e_{zz}(l_x^2 + l_y^2) + B_{33}e_{zz}l_z^2 \\
&\qquad + 2B_{44}l_z(e_{xz}l_x + e_{yz}l_y) + 2B_{66}e_{xy}l_xl_y.
\end{aligned}
\tag{1}
$$

For the AMF with the even magnetic structure $\bar{1}^-4_z^+2_d^- \equiv \bar{1}^-4_z^+2_x^-$ we have:

$$F_{mp} = -2M_0[\gamma_2(l_xP_x + l_yP_y)m_z + \gamma_3(m_xP_x + m_yP_y)l_z + \gamma_4(l_xm_x + l_ym_y)P_z + \gamma_5l_zm_zP_z].$$

For the AMF with the odd magnetic structure $\bar{1}^-4_z^-2_d^- \equiv \bar{1}^-4_z^-2_x^+$ we have:

$$F_{mp} = -2M_0[\gamma_2(l_xP_y + l_yP_x)m_z + \gamma_3(m_xP_y + m_yP_x)l_z + \gamma_4(l_xm_y + l_ym_x)P_z].$$

Here H_E is the exchange interaction field; M_0 is the sublattice saturation magnetization; K and K_2 are magnetic anisotropy constants; E'_l and E'_m are constants of inhomogeneous exchange interaction, $\boldsymbol{H}_m$ is magnetostatic field caused magnetic dipole–dipole interaction, e_{ik}, C_{ik}, B_{ik} are tensor components of strain, elastic and magneto-elastic constants ($i, k = x, y, z$); σ_{ik} is the tensor of external elastic stresses; $\boldsymbol{u}$ is the vector of elastic displacement of the medium elements; $\kappa_\perp$ and $\kappa_\|$ are electropolarization constants. Inasmuch as magneto-electric interaction is determined by the following expression $F_{mp} = -2M_0\gamma_{ikj}l_i m_j P_k$ (where i, j, k are Cartesian coordinates) it is evident that the magneto-electric constants $\gamma_2 \equiv \gamma_{131} = \gamma_{232}$, $\gamma_3 \equiv \gamma_{311} = \gamma_{322}$, $\gamma_4 \equiv \gamma_{113} = \gamma_{223}$, $\gamma_5 \equiv \gamma_{333}$ for an even exchange structure ($\bar{1}^-4_z^+2_d^- \equiv \bar{1}^-4_z^+2_x^-$) and $\gamma_2 \equiv \gamma_{132} = \gamma_{231}$, $\gamma_3 \equiv \gamma_{312} = \gamma_{321}$, $\gamma_4 \equiv \gamma_{123} = \gamma_{312}$ for an odd exchange structure ($\bar{1}^-4_z^+2_d^- \equiv \bar{1}^-4_z^+2_x^+$). Inasmuch as inequality $|\boldsymbol{m}| \ll |\boldsymbol{l}| \ll 1$ takes place, the term with E'_m can be neglected. As for the effects of the demagnetization stipulated by magnetic dipole–dipole interaction, their role will be discussed below.

Let the strength of the magnetic field $\boldsymbol{H}$ be directed along the z-axis, which coincides with the C_4 axis. We shall apply the strength of the electric field $\boldsymbol{E}$ in the easy plane along the 2_d^- axis, i.e. along the bisector of the angle formed by the x- and y-axes. A further detailed examination is conveniently conducted for the case when the constant of the crystallographic magnetic anisotropy in the basal plane xy is positive. The results obtained in this paper will hold for the AFM with both the even and the odd magnetic structures. The unilateral stress σ ($\sigma < 0$ corresponds to compression, whereas $\sigma > 0$ corresponds to expansion) can be directed in the easy plane along or perpendicular to $\boldsymbol{E}$. The direction of $\boldsymbol{E}$ and $\boldsymbol{H}$, as well as the direction of the elastic stress σ are taken so that the linear magneto-electric effect should have the maximal possible manifestation. In the following discussion it will be necessary to rotate the Oxy coordinate system in the basal plane by the angle $\pi/4$, i.e. the x' axis should be directed along 2_d^-.

We shall limit ourselves to the frequency range $\omega \ll \gamma\sqrt{2H_E H_A}$, where γ is the gyromagnetic ratio and $H_A = K/2M_0$ is the magnetic anisotropy field in which the antiferromagnetism vector retains in the easy plane. This frequency range corresponds to magnetic fields significantly weaker than the sublattice-flopping field, which allows us to ignore both the excitation of the branches of the electropolarizing waves spectrum and the excitation of the optical branch of the spin-wave spectrum. At these frequencies, the electrical polarization vector adjusts in a quasi-equilibrium way to magneto-elastic oscillations. We shall also ignore the attenuation of both spin and elastic degrees of freedom in the system, as well as the deviation of the antiferromagnetism vector $\boldsymbol{l}$ from the easy plane, and we shall assume $\boldsymbol{l} = \boldsymbol{l}(\cos\chi, \sin\chi, 0)$, where χ is the angle between the vector $\boldsymbol{l}$ and x'. From the condition of the minimum of free density F with respect to variations of the vectors of electric polarization $\boldsymbol{P}$, of ferromagnetism $\boldsymbol{m}$ and elastic deformations, one can obtain an expression for free-energy density F in an equilibrium state:

$$F = -M_0 H_{ms6}\left(eh\cos\chi + \frac{1}{2}\tau\cos 2\chi - \frac{k_2^*}{8}\cos 4\chi\right) \tag{2}$$

where $H_{ms6} = B_{66}^2/2M_0C_{66}$, $k_2^* = K_2^*/2M_0H_{ms6}$, $K_2^* = K_2 + 2M_0(H_{ms} - H_{ms6})$ and K_2^* is the constant of the crystallographic magnetic anisotropy in the basal plane, renormalized by magnetostriction; $H_{ms} = b^2/[M_0(C_{11} - C_{12})]$ is the magnetostriction field; $e = \gamma_2\kappa_\perp E/H_{ms6}$ is the normalized value of the electric field; $h = H/H_E$ is the normalized value of the magnetic field; $b = B_{11} - B_{12}$; $\tau = \sigma\delta/2B_{66}$. If the stress is directed along $\boldsymbol{E}$, then $\delta = 1$; if it is perpendicular to $\boldsymbol{E}$, then $\delta = -1$, which can be of importance from the point of view of convenience in the performance of the experiment.

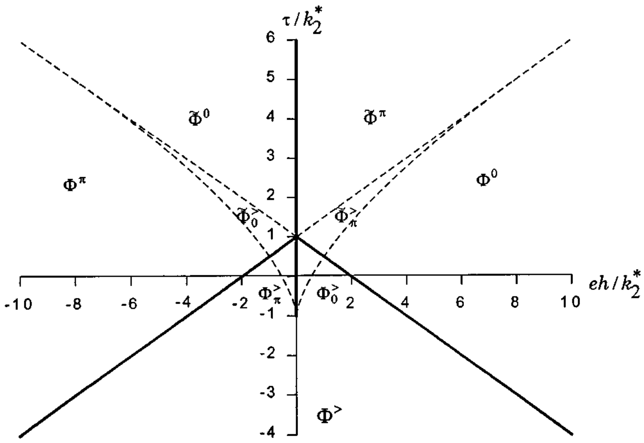

Figure 1. Magnetic phase diagram. Full curves are phase transition lines; broken curves define the borders of the respective metastable phases Φ^0 and Φ^π are symmetrical phases. $\Phi^>$, $\Phi_0^>$ and $\Phi_\pi^>$ are doubly degenerated angular phases.

We shall assume that $K_2^* > 0$. Figure 1 shows the eh–τ diagram of stability of magnetic phases of a tetragonal AFM. As it is seen from figure 1, at $\tau/k_2^* > 1$ the symmetrical phase Φ^0 with $\chi = 0$ is absolutely stable at $\gamma_2 EH > 0$. On the contrary, at $\gamma_2 EH < 0$, the phase Φ^π with $\chi = \pi$ is absolutely stable [2]. The transition between these phases is a phase transition of the first kind. In the case $\tau/k_2^* < 1$ the symmetrical phase Φ^0 is stable in relatively strong electric and magnetic fields. The following inequalities represent the stability conditions of the ground state with $\gamma = 0$:

$$2(\tau - k_2^*) + eh \geqslant 0 \qquad eh > 0. \tag{3}$$

The equality sign in (3) corresponds to the stability loss line of the phase Φ^0. The transition between the symmetrical phase and the doubly degenerated angular phase is a phase transition of the second kind. On the eh–τ plane, the point $eh = 0$, $\tau/k_2^* = -1$ corresponds to the critical point [8]. If $\tau/k_2^* < -1$, then in the angular phase $\Phi^>$ the angle χ continuously varies from zero to π. If $\tau/k_2^* > -1$, then the angular phase $\Phi^>$ splits into two angular phases $\Phi_0^>$ and $\Phi_\pi^>$, in which the angle χ varies from 0 to $(\tau - \varphi_0)/2$ and from $(\pi + \varphi_0)/2$ to π, respectively. The transition between these two angular phases is a phase transition of the first kind. When τ/k_2^* varies from -1 to 1, the jump of φ_0 varies from zero to π. The metastability domains of angular phases $\Phi_\pi^>$ and $\Phi_0^>$ extend to the points $eh/k_2^* = \pm 8$ and $\tau/k_2^* = 5$ on the eh–τ plane. In figure 1, the '$\sim$' (tilde) sign corresponds to metastable phases. The equations for the curves, which separate metastable angular phases from the stable phases: $eh/k_2^* = \pm[2(\tau/k_2^* + 1)/3]^{3/2}$; metastable angular and metastable symmetrical phases: $eh/k_2^* = \pm 2(\tau/k_2^* - 1)$.

The vectors of ferromagnetism $\boldsymbol{m}$ and of electric polarization $\boldsymbol{P}$ have the following components:

$$m_{x'} = m_{y'} = 0 \qquad m_z = \frac{H + \gamma_2 \kappa_\perp E \cos\chi - \dot{\chi}/\gamma}{2H_E}$$

$$P_{x'} = \kappa_\perp E + \frac{M_0 \gamma_2 \kappa_\perp}{H_E}(H - \dot{\chi}/\gamma)\cos\chi \qquad P_{y'} = \frac{M_0 \gamma_2 \kappa_\perp}{H_E}(H - \dot{\chi}/\gamma)\sin\chi$$

$$P_z = 0.$$

Returning to a discussion of the role of the long-range dipole–dipole magnetic interactions, at first we notice that the interactions mentioned are insignificant in ATMs. Second, $\boldsymbol{H}_m = 0$ supposing that the wave propagates along the axis x'. This is evident from the analysis of magnetostatic equations, a geometry of the problem and above expressions for $m_{x'}$, $m_{y'}$ and m_z. For the sample bounded along the z-axis the energy of magnetic dipole–dipole interaction can be reduced to the form $16\pi M_0^2 m_z^2$. This brings to the substitution $H_E \to H_E + 4\pi M_0$ (however, $H_E \gg 4\pi M_0$).

Let us consider nonlinear excitations relative to the ground state with $\chi = 0$, the said excitations propagating along the x' axis. We shall hereafter omit the $'$ (prime) sign in coordinates. The density of the Lagrange function has the form as follows:

$$L = \frac{M_0}{2\gamma^2 H_E}(\dot{\chi}^2 - c^2\chi_x^2) + M_0 H_{ms6}\left(eh\cos\chi + \frac{1}{2}(1+\tau)\cos 2\chi - \frac{1}{8}k_2\cos 4\chi\right)$$
$$-\frac{b}{2}u_{xy}\sin 2\chi - \frac{B_{66}}{2}u_{xx}\cos 2\chi - \frac{\rho}{2}(S_1^2u_{xx}^2 + S_{44}^2u_{xz}^2 + S_2^2u_{xy}^2 - \dot{\boldsymbol{u}}^2).$$

Here ρ is the medium density, $u_{xi} = \partial u_i/\partial x$ $(i = x, y, z)$ are the deviations of the deformation tensor from the value in the ground state [9]; $c^2 = \gamma^2 E_l' a_0^2 H_E/M_0$ is square of the minimal phase velocity of the spin wave; $S_{44}^2 = C_{44}/\rho$; $S_1^2 = (C_{11} + C_{12} + 2C_{66})/2\rho$, $S_2^2 = (C_{11} - C_{12})/2\rho, k_2 = K_2/2M_0H_{ms6}$ is the dimensionless constant of the crystallographic magnetic anisotropy in the basal plane (not renormalized by a magneto-elastic interaction).

We shall write out the Euler–Lagrange equations, in which nonlinear terms to the third order for small deviations χ and u_{xi} are retained:

$$(\partial_t^2 - S_2^2\partial_x^2)u_{xy} - \frac{b}{\rho}\partial_x^2\chi + \frac{2b}{3\rho}\partial_x^2\chi^3 = 0$$
$$(\partial_1^2 - S_1^2\partial_x^2)u_{xx} + \frac{B_{66}}{\rho}\partial_x^2\chi^2 = 0$$
$$(\partial_t^2 - S_{44}^2\partial_x^2)u_{xz} = 0$$
$$(\partial_t^2 - c^2\partial_x^2 + \omega_0^2)\chi + \frac{b\gamma^2 H_E}{M_0}u_{xy} - \frac{\gamma^2 H_E}{M_0}$$
$$\times\left\{2B_{66}u_{xx}\chi + 2bu_{xy}\chi^2 + \frac{1}{6}M_0H_{ms6}[eh + 8(1+\tau) - 32k_2]\chi^3\right\} = 0. \qquad (4)$$

Here ω_0 is a gap in the spectrum of the low-frequency branch (in this case quasi-ferromagnet branch, see below) of the spin waves in the AFM:

$$\omega_0^2 = \gamma^2\left[\gamma_2\kappa_\perp EH + 2H_E H_{ms6}\left(\tau - k_2^* + \frac{H_{ms}}{H_{ms6}}\right)\right]$$
$$= \gamma^2 H_E H_{ms6}\left[eh + 2\left(\tau - k_2^* + \frac{H_{ms}}{H_{ms6}}\right)\right].$$

The minimal value of the frequency ω_0 in the point of the phase transition determines the so-called magneto-elastic gap for magnons [9]:

$$\omega_{0\,\min} = \omega_{ms} = \gamma\sqrt{2H_E H_{ms}}.$$

The dispersion equation for linear magneto-elastic waves is determined by the equation below:

$$(\omega^2 - S_1^2k^2)(\omega^2 - S_{44}^2k^2)[(\omega^2 - S_2^2k^2)(\omega^2 - \omega_0^2 - c^2k^2) - \omega_{ms}^2S_2^2k^2] = 0.$$

Its solution yields the following branches of the spectrum [9]:

(a) the quasi-ferromagnetic branch

$$\omega_1^2 = \omega_0^2 + fk^2$$

(b) the quasi-acoustic branches

$$\omega_2^2 = S^2k^2 + rk^4 \qquad \omega_3^2 = S_1^2k^2 \qquad \omega_4^2 = S_{44}^2k^2$$

where

$$f = c^2 + \frac{\omega_{ms}^2}{\omega_0^2}S_2^2 \qquad S^2 = S_2^2\left(1 - \frac{\omega_{ms}^2}{\omega_0^2}\right) \qquad r = \frac{\omega_{ms}^2}{\omega_0^4}(c^2 - S^2)S_2^2. \tag{5}$$

When approaching the point of the phase transition of the first kind from the symmetrical phase Φ^0 into Φ^π (for example, by decreasing the electric field value) the speed S of the transverse sound reduced to zero. On the other hand, in AFMs with a low Néel temperature, a situation is possible, when $S > c$. When approaching the phase transition point this condition may be violated, that will mean the reversal of the sign of the dispersion r of the transverse quasi-acoustic mode when the strengths of the electric and the magnetic fields decrease.

3. Existence domain of quasi-acoustic solitons

For examining the interaction of quasi-acoustic waves it is necessary to assume that magnetization oscillations adjust in a quasi-equilibrium manner to elastic deformations. This means that activation waves are not excited, thus permitting the use of the version of the reductive theory of perturbations, this version being based on coordinate expansion [10]. As a result, for the transverse component of the deformation tensor $U \equiv u_{xy}$ we shall obtain the MKdV:

$$2S\frac{\partial U}{\partial t} - r\frac{\partial^3 U}{\partial \zeta^3} + q\frac{\partial}{\partial \zeta}U^3 = 0.$$

$$q = \frac{5b^4\gamma^8 H_E^4 H_{ms6}}{2\rho M_0^3\omega_0^8}\left[eh + \frac{8}{5}\left(\tau + \frac{S_1^2 - S^2 - S_{66}^2}{S_1^2 - S^2}\right)\right]. \tag{6}$$

Here $\zeta = x - St$. The remaining variables are expressed in terms of U in the following manner:

$$\chi = -\frac{b\gamma^2 H_E}{M_0\omega_0^2}U \qquad u_{xx} = \frac{b^2 B_{66}\gamma^4 H_E^2}{\rho M_0^2(S_1^2 - S^2)\omega_0^4}U^2.$$

If the condition $rq < 0$ is satisfied [5], then the MKdV (6) has soliton solutions. The single-soliton solutions is determined by the formula [10]:

$$U = \frac{A}{ch(\xi/\Delta)} \tag{7}$$

where

$$A = 2\sqrt{\frac{S\lambda}{q}} \qquad \Delta = \sqrt{\frac{|r|}{2S\lambda}}$$

$\xi = x - x_0 - (S + \lambda)t$; $\lambda > 0$ and x_0 are material parameters. The parameter λ is equal to the difference between the soliton velocity and the speed of sound S. The applicability of the perturbations theory is conditioned by $\lambda \ll S$. In the case under consideration $q > 0$, and

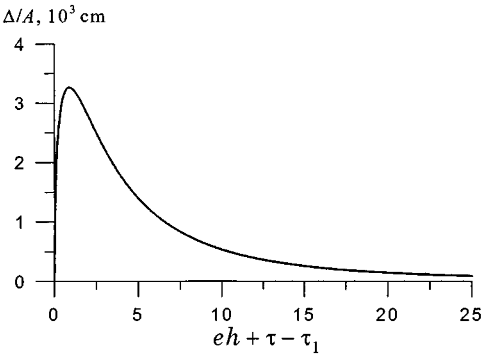

Figure 2. Dependence on $\tau - \tau_1 + eh$ of the ratio of the MKdV soliton width Δ to its amplitude A.

solitons do exist, if $r < 0$, i.e. $S > c$. This condition, taking into account (5), can be rewritten as:

$$\frac{\gamma_2 \kappa_\perp E H}{H_E} + \left(\frac{\sigma \delta H_{ms6}}{B_{66}} - \frac{K_2^*}{M_0} \right) > \frac{2 H_{ms} c^2}{S_2^2 - c^2} \qquad S_2 > c.$$

In the case $\tau < \tau_1$, where

$$\tau_1 = k_2^* + \frac{H_{ms} c^2}{H_{ms6}(S_2^2 - c^2)}$$

the soliton can exist only above a certain critical value eh, equal to $2(\tau_1 - \tau)$, and *vice versa,* at $\tau \geqslant \tau_1$, the soliton can exist at $eh > 0$. Therefore, if a stress is applied which is somewhat smaller than τ_1, it is possible to control the soliton stability using the weak electric and magnetic fields. In the case of $\tau \geqslant \tau_1$, due to a change of the phase Φ^0 stability domain, only sufficiently strong elastic and magnetic fields can affect the soliton existence.

Figure 2 presents the graph illustrating the dependence of the ratio of the soliton width Δ to its amplitude A upon stress, electric and magnetic fields. If $\tau - \tau_1 + eh$ tends to zero, then $r \to 0$. Therefore, the soliton width Δ also tends to zero. The soliton amplitude never reaches zero. In spite of the fact that the reliable experimental evidences for observation of soliton states in the investigated low-symmetric AFMs are lacking at present, nevertheless we cite certain numerical estimates which give strong evidence for a possibility of a detection of the above indicated interesting effects. In our opinion for this purpose trirutiles of Cr_2TeO_6 and V_2WO_6 type [11–13], rare-earth phosphates and vanadates—compounds of the $HoPO_4$ and $GdVO_4$ type [14–16]—are promising materials. We used the typical values of the crystal parameters for numerical estimates: $M_0 \approx 150$ Oe; $H_E \approx 150$ kOe; $\rho \approx 5$ g cm^{-3}; b, $B_{66} \approx 5 \times 10^6$ erg cm^{-3}; $C_{11} \approx 2 \times 10^{12}$ erg cm^{-3}; $C_{12} \approx 0.9 \times 10^{12}$ erg cm^{-3}; $C_{66} \approx 0.5 \times 10^{12}$ erg cm^{-3}; $c \approx 2 \times 10^5$ cm s^{-1}; $\lambda \approx 100$ cm s^{-1}; $k_2^* \approx 30$ (this corresponds to $K_2 \approx 10^3$ erg cm^{-3}); $\gamma_2 \kappa_\perp \approx 0.3$ (this corresponds to magneto-electric susceptibility $\alpha \approx 3 \times 10^{-4}$; in this case the measurement unit of the standardized value of the electric field e in SI corresponds approximately to 10 kV m^{-1}). At $h = 0.05$ ($H = 7.5$ kOe), $\tau = 35$ ($\sigma = 35$ MPa) and $e = 200$ ($E = 20$ kV cm^{-1}), the soliton parameters are the following: amplitude $\sim 2 \times 10^{-6}$, width $\sim 4 \times 10^{-4}$ cm, propagation velocity $\sim 3 \times 10^5$ cm s^{-1}, the amplitude of angular oscillations χ of the AFM vector is about 0.03.

The obtained results are true if the conditions are satisfied of the applicability of the system of equations in (4), i.e. $\Delta \gg a_0$ (a_0 is the crystal's lattice constant) and $|\boldsymbol{m}| \ll |\boldsymbol{l}| \approx 1$. At $r \to 0$, these conditions are violated. On the other hand, at $r \to 0$, within the framework of the examined field geometry, the transverse elastic wave u_y becomes dispersionless, and the solitons of the (7) type do not exist. As has been shown above, with the help of the electric and magnetic fields one can achieve $r \to 0$, i.e. the disappearance of the indicated type of solitons.

4. Existence domain of activation solitons and of Zakharov–Benney magneto-elastic resonance

In the case when the coupling between gapless and activation waves (see second and fourth equations of system (4)) is quadratic in magnetization, one can expect that the beating in the activation wave will excite the modes with zero activation energy (gapless or Goldstone's modes) [17]. This process is a resonance process and was first discovered by Benney. The corresponding system of nonlinear equations was first obtained by Zakharov [18], whereas in [19, 20] it has been shown that it is integrable by the method of the inverse scattering problem. When Zakharov–Benney resonance originates in the AFM, the short-wave quasi-ferromagnetic modes will excite long-wave acoustic modes.

For the purpose of examining the interaction of activation waves, as well as of the interaction of activation and Goldstone's waves in the vicinity of Zakharov–Benney long- to short-wavelength resonance, in the system of equations (4) it is necessary to pass over from the dynamic variables χ and u_{xy} to normal variables φ and u_1. It is necessary then to assume that $u_1 = 0$ and to average the thus obtained system of equation with respect to the fast spatial and temporal oscillations, choosing $\varphi = \Psi \exp[\mathrm{i}(\omega_1 t - kx)] + \mathrm{cc}$ [6]. As a result we shall arrive at the system of equations:

$$(\partial_t^2 - S_1^2\partial_x^2)u_{xx} + \frac{2B_{66}}{\rho}\partial_x^2|\Psi|^2 = 0$$

$$\mathrm{i}\partial_t\Psi - \frac{f}{2\omega_1}\partial_x^2\Psi + \mathrm{i}c_1\partial_x\Psi - \frac{B_{66}\gamma^2 H_E}{M_0\omega_1}u_{xx}\Psi - \frac{1}{4}\frac{\omega_5^2}{\omega_1}|\Psi|^2\Psi = 0 \qquad (8)$$

where

$$\omega_5^2 = \gamma^2 H_E H_{ms6}[eh + 8(\tau - 3 - 4k_2^* + 4H_{ms}/H_{ms6})].$$

In the system of equations (8), if we pass over to the coordinate system [10] which moves with the group velocity of activation modes ($\xi = x - c_1 t$; $c_1^2 \neq S_1^2$; $c_1 \equiv \partial\omega_1/\partial k$) we can obtain the NSE:

$$\mathrm{i}\partial_t\Psi - \frac{f}{2\omega_1}\partial_\xi^2\Psi - g_0|\Psi|^2\Psi = 0$$

$$g_0 = \frac{\gamma^2 H_E H_{ms6}}{4\omega_1}\left[eh + 8\left(\tau - 3 + \frac{4H_{ms}}{H_{ms6}} - 4k_2^* + \frac{2S_{66}^2}{S_1^2 - c_1^2}\right)\right].$$

At $g_0 > 0$, according to Lighthill's criterion [10], linear activation waves become modulationally unstable as regards formation of the envelope solitons—the bions. At small values of the wave vector ($c_1 \to 0$), the condition of the envelope solitons existence can be described by the expression:

$$\frac{\gamma^2\kappa_\perp EH}{H_E} + 8\left(\frac{\sigma\delta H_{ms6}}{2B_{66}} + 4H_{ms} - \frac{2K_2^*}{M_0} - \frac{3S_1^2 - 2S_{66}^2}{S_1^2}H_{ms6}\right) > 0. \qquad (9)$$

If $\tau < \tau_2$, where

$$\tau_2 = 4\left(k_2^* + \frac{3S_1^2 - 2S_{66}^2}{4S_1^2} - \frac{H_{ms}}{H_{ms6}}\right)$$

then the bion can exist only above some critical value of eh equal to $8(\tau_2 - \tau)$ and, *vice versa*, at $\tau \geqslant \tau_2$, the bion can exist at $eh > 0$. If, therefore, a stress is applied which is somewhat smaller than τ_2, the bion's stability can be controlled with the help of weak electric and magnetic fields. If k_2^* is sufficiently great, then τ_2 can turn out to be very great and thus unattainable in actual conditions. The condition (9) will then be violated. In this case the examination should be conducted in the conditions as follows: $\boldsymbol{E}$ is directed along the former unaccented x-axis for the odd magnetic structure and along the unaccented y-axis for the even magnetic structure, whereas the stress σ is directed along or perpendicular to $\boldsymbol{E}$. If such is the case, the condition of bion existence similar to (9) will be fulfilled.

The bion solution of the NSE can be presented as [10]

$$\Psi = \frac{d_0 \exp[\mathrm{i}(k_0\xi - \omega_6 t)]}{ch[(1/\Delta_1)(\xi + ut)]}$$

where

$$\Delta_1 = \sqrt{\frac{f}{\omega_1 g_0 d_0^2}} \qquad k_0 = \frac{v\omega_1}{f} \qquad \omega_6 = \frac{1}{2}\left(g_0 d_0^2 - \frac{v^2\omega_1}{f}\right).$$

At $h = 0.05$ ($H = 7.5$ kOe), $\tau = 120$ ($\sigma = 120$ MPa) and $e = 200$ ($E = 20$ kV cm^{-1}), the soliton velocity with respect to the speed of the longitudinal sound $v \approx 100$ cm s^{-1}, amplitude $d_0 \approx 0.05$, the soliton parameters are the following: width $\Delta_1 \sim 5 \times 10^{-4}$ cm; $k_0 \sim 70$ cm^{-1}; $\omega_6 \sim 3 \times 10^6$ s^{-1} ($\omega_6/\omega_0 \sim 10^{-4}$). For these calculations the above-mentioned values of crystal parameters (see section 3) have been used.

Magneto-elastic long–short wavelength Zakharov–Benney resonance is realized when the phase velocity of dispersionless quasi-acoustic wave is equal to the group velocity of the activation wave [10]:

$$S_1 = c_1 \equiv \frac{\partial\omega_1}{\partial k}. \tag{10}$$

When the condition (10) is satisfied, the system of equations (8) boils down to the integrable system of Zakharov–Benney [10]:

$$\begin{cases} \mathrm{i}\dfrac{\partial\Psi}{\partial t} - a_1\dfrac{\partial^2\Psi}{\partial\xi^2} - a_2 u_{xx}\Psi = 0 \\[2mm] \dfrac{\partial u_{xx}}{\partial t} - a_3\dfrac{\partial|\Psi|^2}{\partial\xi} = 0. \end{cases}$$

Here $a_1 = f/2\omega_1$, $a_2 = \gamma^2 H_E B_{66}/(M_0\omega_1)$, $a_3 = B_{66}/(\rho S_1)$ and $\xi = x - c_1 t$.

Equality (10) can only be satisfied when the inequality $S_1^2 < f$ holds, i.e.

$$\pm\left[\frac{\gamma_2\kappa_\perp EH}{H_E} + \left(\frac{\sigma\delta H_{ms6}}{B_{66}} - \frac{K_2^*}{M_0}\right)\right] > \pm 2\frac{S_2^2 + c^2 - S_1^2}{S_1^2 - c^2} H_{ms}. \tag{11}$$

Here the + sign corresponds to the case $c > S_1$, whereas the $-$ sign corresponds to the case $c < S_1$. Thus, at $S_1 < c$, i.e. when the minimal phase velocity of spin waves is greater than the velocity of the longitudinal sound, the Zakharov–Benney resonance, as is seen from (11), exists in the stability area of the phase Φ^0 and *vice versa*, at $S_1 > c$, two cases are possible. If

$$S_1^2 > S_2^2 + c^2$$

no resonance exists. When the inequalities

$$c^2 < S_1^2 < S_2^2 = c^2 \tag{12}$$

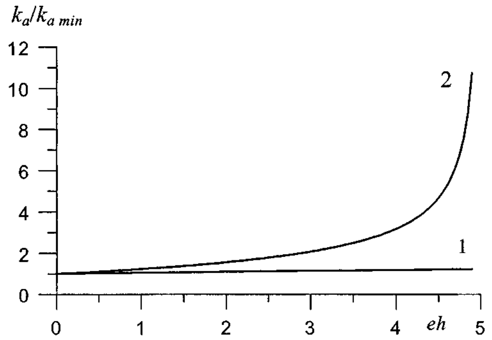

Figure 3. Dependence of the resonance value of the wave vector of the spin wave on electric and magnetic fields at the stress value $2(\tau_3 - \tau) = 5$: 1, at $S_1/c \approx 3.5$; 2, at $S_1/c \approx 1.05$.

are satisfied, the resonance can exist in the limited area of varying stress, electric and magnetic fields. In this case if stress $\tau \geqslant \tau_3$, where

$$\tau_3 = k_2^* + \frac{H_{ms}}{H_{ms6}} \frac{S_2^2 + c^2 - S_1^2}{S_1^2 - c^2}$$

then no resonance exists. If $\tau < \tau_3$, the values of the electric and of the magnetic fields at which the resonance does exist are limited:

$$eh < 2(\tau_3 - \tau).$$

From the condition (10) the resonance wave number of the activation wave is obtained:

$$k_a = \frac{S_1\omega_0}{\sqrt{f(f - S_1^2)}}. \tag{13}$$

In the case (12) at $eh \to 2(\tau_3 - \tau)$ the value of f tends to S_1^2, i.e. the expression in the denominator (13) tends to zero, and the wave vector of the activation wave $k_a \to \infty$. This means physically that the group velocity of the activation wave cannot be compared with the phase velocity of Goldstone's wave at any value of the wave vector k, and the condition (10) is not satisfied. Therefore, at $eh \approx 2(\tau_3 - \tau)$ the resonance vanishes. Figure 3 shows the dependence of the resonance value of the wave vector of the spin wave k_a on the electric and magnetic fields at $2(\tau_3 - \tau) = 5$. For this case the minimal phase velocity of the spin waves was taken as: $c \approx 6 \times 10^5$ cm s^{-1} ($S_1/c \approx 1.05$), whereas the rest of the crystal's parameters were similar to those specified in section 2. The analysis shows that if the condition (12) is satisfied (the AFM with a low Néel temperature), the resonance value of the wave vector of the spin wave k_a can be changed by an order of the magnitude. If $c > S_1$, then k_a is hardly affected by the electric and magnetic fields.

All the results obtained in this paper are also true for the case when the constant of the crystallographic magnetic anisotropy in the basal plane, which has been renormalized by magnetostriction, is less than zero only for other orientations of the electric field $\boldsymbol{E}$ and stresses σ. In the initial coordinate systems (see (1)), the electric field should be directed along the x-axis for the odd magnetic structure, and along the y-axis for the even magnetic structure, whereas the stress—in the basal plane along or perpendicular to the electric field, and the y-axis should be taken as the direction of the wave propagation. In this case, only the constants in the formulae of this paper will have to be modified: B_{66} should be replaced with b, b should be replaced with $(-B_{66})$, S_1—with S_{11}, $S_2 \leftrightarrow S_{66}$, $H_{ms6} \leftrightarrow H_{ms}$, k_2^* with $(-k_2^*)$, and in the expression for τ, $2B_{66}$ should be replaced with b.

5. Conclusion

The results obtained in this study enable one to make a conclusion that the electric field $\boldsymbol{E}$ is another regulator, along with magnetic field $\boldsymbol{H}$ and mechanical stress σ, of both the resonance and non-resonance nonlinear processes, which may occur in easy-plane AFMs. The effect of $\boldsymbol{E}$ and $\boldsymbol{H}$ is especially pronounced in the vicinity of orientational phase transitions, but for the latter to be achieved, the values $\boldsymbol{E}$ and $\boldsymbol{H}$ may have to be quite great. To achieve the condition of the orientational phase transition, the uniaxial directed stress σ can be used. For the MKdV solitons, the NSE bions and the Zakharov–Benney solitons, the critical values of stresses have been identified at which the influence of the magneto-electric effect is most strongly pronounced.

Since the MKdV soliton stability is determined by the sign of the difference in velocity between a transverse sound wave and the minimal phase velocity of spin waves, in AFMs with a low Néel temperature the sign of this difference can be reversed using the electric and magnetic fields, thus effectively controlling stability of magneto-elastic solitons described by the MKdV.

The stability of the bionic solution of the NSE is determined by the sign of the self-action coefficient of the activation modes. The paper has demonstrated that using the electric and magnetic fields this sign can be reversed. For the AFMs with a low Néel temperature, in which inequalities (12) are satisfied, the resonance may exist within the limited domain of varying stress, electric and magnetic fields. This domain can be transcended with the help of the electric and magnetic fields.

As is already mentioned above (see section 3), the following AFMs may display the effects as discussed above: trirutiles of the Cr_2TeO_6, V_2WO_6 type [11–13] rare-earth phosphates and vanadates—compounds of the $HoPO_4$ and $GdVO_4$ type [14–16]. At the present time the details of the linear spectrum of the spin waves and the conditions of their excitation for example Gd_2CuO_4 [21] are known. Also, convincing evidences for the presence of the magneto-electric effect in R_2CuO_4 [R $=$ Gd, Nd or Sm] [22] are present. Moreover, as is shown above the low-symmetric (tetragonal) AFMs are superb model media for investigation of essential nonlinear magneto-elastic phenomena. The effects discussed above are especially important in the vicinity of the points of spin-reorientation phase transitions. It is to be hoped that the effects predicted in this paper will be discovered in the near future.

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